Chance and probability

S3/S4 (Ages 14–16) — Core lesson with separate Quantum Physics extension

Curriculum links: MNU/MTH 3-22a (chance and uncertainty); MNU 4-22a (link between probability and expected frequency); National 4/5 Applications of Mathematics — Numeracy unit (comparing relative frequencies; calculating probability from relative frequency as a fraction, decimal or percentage).

Background

Quantum physics is the branch of physics that describes how nature behaves at the smallest scales, such as atoms, electrons and particles of light called photons. At this level, the familiar rules of everyday physics no longer give the whole picture. Instead, particles can behave in ways that seem very strange compared with objects we experience in everyday life.

One of the most surprising ideas in quantum physics is **superposition**. A quantum object such as an electron can exist in a combination of different possible states at the same time, until a measurement is made. For example, an electron can have a combination of different possible "spin" states rather than having just one definite state. This does not simply mean that we do not know which state it is in; its behaviour genuinely includes several possibilities at the same time. But when a measurement is made, we only find one particular state. A useful analogy for superposition is a spinning coin: while it's spinning, we can describe it in terms of the probabilities of it landing heads or tails, and when it stops we see just one result. A spinning coin is not actually a quantum superposition, but the analogy can help us understand some of the mathematics involved.

Another remarkable idea is **entanglement**. Two quantum particles can become linked so that their properties are described by a single quantum state, even when they are a very long way apart. Measuring one particle can then tell us something about the result we will get from measuring the other. The correlations between entangled particles can be much stronger than is possible with ordinary objects. Imagine tossing two coins and knowing that if one is heads, the other must also be heads, no matter how far apart they are.

This lesson looks at the mathematics of spinning coins and considers whether it's possible to reproduce some of the behaviour associated with quantum superposition and entanglement.

Equipment

- Coins for pupils to toss.
- Copies of the record sheet.

Success criteria at the end of the lesson

- I can predict the likely outcome of a chance event and explain my reasoning.
- I can carry out a simple experiment involving chance and record my results accurately.
- I can compare my experimental results with my prediction and explain any difference.
- I can calculate probability from relative frequency and express it as a percentage. (National 4/5 Numeracy)
- I can explain, using an example from today's lesson, how increasing the number of trials brings relative frequency closer to theoretical probability. (MNU 4-22a)

Core lesson: Chance and probability

Ages 14–15 (S3) and 15–16 (S4). Use the recording sheet to capture results as you go so.

Task 1 — Coin toss, individual then paired

- Choose a partner.
- In a minute you will each toss a coin 20 times. What percent of the time do you think you will get heads and what percent tails?
- Do the experiment and write down what you actually get.
- Overall, what percentage of heads and tails did you get? How close was this to your prediction?

- Add your results to your partner's. What percentage of heads and tails did you get when your results were combined? How does it compare to your prediction now? Why is that?
- What would happen if everyone in the class combined their results? How would it compare to your prediction then?

Task 2 — How often do two coins agree?

- You are going to do another 20 heads or tails experiments in your pair, but this time you aren't going to write whether they are heads or tails. Instead, record whether the two coins showed the same side, or different (regardless of whether they were heads or tails).
- What percent of the time do you think you will get the same result?
- What percent of the time did you actually get the same result? How does this compare with what you expected?

Task 3 — Rigging the odds

- You are now going to play a game. You are going to sit back-to-back with your partner, and silently choose heads or tails (without actually tossing a coin), then write down what you chose. You will repeat this 20 times. The aim of the game is to choose the same as your partner more than half of the time. There are two rules: 1. You are not allowed to talk or signal to each other after the game begins. 2. Your individual results should look random, with no pre-agreed patterns.
- Working together, try to think of a strategy the two of you could use that lets you win this game.
- What results do you think you will get?
- Carry out the experiment and record the results.
- Was there any way of doing it without breaking the rules?
- If you think your strategy worked, do you think it would still work for a much longer game with a lot more coin tosses?

Final task — Whole-class discussion

- One pair, who is confident they have a strategy for increasing the number of times they both choose the same result stands in front of the whole class to demonstrate their strategy.
- The class discusses whether the strategy really works.
- If no pair has a strategy that they think works, discuss why the task seems impossible.
- Discuss what it means for the individual results to "look random" - with no repeating patterns. What does "random" mean? This is one of the things that makes the game really hard. Even if you simply cheat by agreeing a random-looking set of results in advance – if you kept going you would eventually run out of agreed answers.
- It seems that to be absolutely sure of winning this game, no matter how long it lasts, you must be able to communicate after the game starts. But entangled particles seem able to win a similar game, without communicating with each other - even if they are a huge distance apart.

Extension

The discussion below goes beyond S3/S4 Numeracy and Mathematics and sits closer to Higher/Advanced Higher Physics content. Treat it as enrichment: flag it clearly to pupils as 'not assessed today', and don't use it as evidence against the Success criteria above.

Refer to the "Superposition" leaflet.

Vocabulary to introduce

- Superposition - a quantum system existing in a combination of states until measured.
- Measurement - the act of observing a quantum system, which forces it into one definite state.
- Entanglement - a kind of correlation between particles that means the result of measuring one is affected by the result of measuring the other - no matter how far apart they are.

Discussion questions

- What can probability tell us about things we can't predict individually?

  Probability reveals the structure hidden inside randomness. Probability gives us a way to understand patterns in situations where individual outcomes are completely unpredictable. Even if each event - a coin flip, a radioactive decay,

a person's daily choices - is uncertain on its own, the distribution of many such events often settles into a stable, measurable shape. The more events there are, the more closely they match the prediction.

- How is quantum superposition different from a spinning coin?

  We have used the analogy of a spinning coin to help understand quantum superposition, but actually it is not the same. Quantum effects only exist at very small scales, like subatomic particles. In reality a spinning coin is always physically either heads or tails; we just cannot see which. A quantum particle can genuinely exist in a superposition of different states at the same time, and this is used in technologies like quantum computers.

- Why isn't entanglement just two particles having agreed their answers in advance?

  Entanglement isn't just two particles secretly agreeing on answers because the correlation between them only appears when you choose what to measure, and those choices can be made after the particles have separated. This strange behaviour has been found experimentally, and it is now used in technologies like quantum cryptography.

Optional follow-up (if used as an extension task)

- Pupils research one real technology that uses entanglement (e.g. quantum cryptography, quantum computing) and summarise in 2–3 sentences what problem it solves.
- Pupils write one sentence explaining, in their own words, why the coin analogy is not the same as superposition.

Recording sheet

	Prediction (%H / %T)	Actual tally (H / T)	Actual percent (% H / % T)	How does the result compare to your prediction? Why?
Task 1 (individual)				
Task 1 (pair combined)				
	Prediction (%same / %different)	Actual tally (same / different)	Actual percent (% same / % different)	How does the result compare to your prediction? Why?
Task 2 (pair)				
	Prediction (%same / %different)	Actual tally (same / different)	Actual percent (% same / % different)	How does the result compare to your prediction? Why?
Task 3 (pair)				